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**DISCUSSION PAPERS IN ECONOMICS, FINANCE  
AND INTERNATIONAL COMPETITIVENESS**

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of Non-Linearity**

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ISSN 1324-5910

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***Discussion Paper No. 168, December 2003***

Series edited by  
Associate Professor Andrew C Worthington

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**School of Economics and Finance**

# A time-domain test for some types of non-linearity

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December 10, 2003

## Abstract

The bispectrum and third-order moment can be viewed as equivalent tools for testing for the presence of non-linearity in stationary time series. This is because the bispectrum is the Fourier transform of the third order moment. An advantage of the bispectrum is that its estimator comprises terms which are asymptotically independent at distinct bifrequencies under the null hypothesis of linearity. An advantage of the third order moment is that its values at any subset of joint lags can be used in the test, whereas when using the bispectrum the entire (or truncated) third order moment is required to construct the Fourier transform. In this paper we propose a test for non-linearity based upon the estimated third order moment. We use the phase scrambling bootstrap method to give a non-parametric estimate of the variance of our test statistic under the null hypothesis. Using a simulation study we demonstrate that the test obtains its target significance level, with large power, when compared to an existing standard parametric test that uses the bispectrum. Further we

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<sup>†</sup>The authors would like to thank Dimitris Kugiumtzis for his code to calculate the CAAFT and Simon Sando for help in preparing this manuscript. RCW thanks the Principal and Fellows of Jesus College, Oxford, for a Visiting Senior Research Fellowship, during the tenure of which this work was completed.

show how the proposed test can be used to identify the source of non-linearity due to interactions at specific frequencies. We also investigate implications for heuristic diagnosis of non-stationarity.

**Some key words:** Third-order moment, bispectrum, non-linear, non-stationary, time series, bootstrap, phase scrambling.

# 1 Introduction

## 1.1 Motivation

For the purposes of this paper, let  $\{X_t\}_{t=1}^{\infty}$  be a zero mean, ergodic, discrete time series, with third order moment given by

$$\mu(r, s) = E(X_t X_{t+r} X_{t+s}) \quad (r, s \text{ integers}) \quad (1)$$

The bispectrum is the double Fourier transform of the third order moment, given by

$$b(\omega_j, \omega_k) = \sum_{r=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} \mu(r, s) \exp\{-i(\omega_j r + \omega_k s)\} \quad (-\pi \leq \omega_j, \omega_k \leq \pi). \quad (2)$$

The quantities in (1) and (2) contain essentially the same information. The time-bicoherence was proposed by [1] and is defined to be

$$B_t(r, s) = \frac{\sum_{r'=-\infty}^{\infty} \sum_{s'=-\infty}^{\infty} \mu(r', s') \mu(r + r', s + s')}{\sum_{k=-\infty}^{\infty} \gamma(k) \gamma(r + k) \gamma(s + k)} \quad (3)$$

where  $\gamma(k) = E(X_t X_{t+k})$  is the second order moment or autocovariance function of the given time series. The normalised bispectrum (frequency-bicoherence) was proposed by [2] and is defined as

$$B_f(\omega_j, \omega_k) = \frac{|b(\omega_j, \omega_k)|^2}{s(\omega_j) s(\omega_k) s(\omega_j + \omega_k)} \quad (4)$$

where  $s(\omega)$  is the spectrum, using the classical Wold representation given by the Fourier transform of the autocovariance function. We define a general linear series

$$X_t = \sum_{j=1}^{\infty} \beta_j \varepsilon_{t-j} + \varepsilon_t \quad (t = 1, 2, \dots), \quad (5)$$

where  $\sum_{j=1}^{\infty} \beta_j^2 < \infty$ , and  $\{\varepsilon_t\}$  is a sequence of independent and identically distributed variables (i.i.d.) variables with zero mean and a finite constant variance,  $\sigma_\varepsilon^2$ .

Both  $B_t(r, s)$  and  $B_f(\omega_j, \omega_k)$  are constant everywhere for the linear series (5) and equal to  $\{E(\varepsilon_t^3)\}^2 / \sigma_\varepsilon^6$ . For a linear series with a symmetric error distribution we have that  $E(\varepsilon_t^3) = 0$ , and so both  $B_t(r, s)$  and  $B_f(\omega_j, \omega_k)$  are also useful for testing whether or not a series is Gaussian white noise. The advantage of the frequency-bicoherence as a test statistic for non-linearity is that the two quantities  $\hat{B}_f(\omega_j, \omega_k)$  and  $\hat{B}_f(\omega_{j'}, \omega_{k'})$  ( $(j, k) \neq (j', k')$ ,  $(j, k, j', k') \in \Delta = \{(j, k); 0 < \omega_j < \pi, 0 < \omega_k < \omega_j, \omega_j + \frac{1}{2}\omega_k < \pi\}$ ), are asymptotically independent at a rate  $o(n)$  for a linear series when using the so-called direct bispectrum estimate [3]. As these independent quantities also have an asymptotic complex Gaussian distribution (under the null hypothesis of linearity), a test can be formed with a desired size. This was the basis of the test detailed in [4], but this method has been shown to have low power [5]. Further [6] showed that the assumption that the test statistic converges to a non-central Chi-squared distribution is violated for the particular bispectrum estimate used.

The advantage of using the time-domain third order moment to test for non-linearity over using the bispectrum is one of simplified computation, but even for a linear stationary series there are correlated values of  $\hat{\mu}(r, s)$  in the region  $0 \leq r \leq s < \infty$ . This complicates the construction of a parametric test. In the present paper we overcome this problem by using the phase scrambled bootstrap procedure to give a non-parametric estimate of the variance in the third order moment under the assumption of linearity. Another advantage of using the third order moment over the bispectrum is that we can exclude the estimate at  $\mu(0, 0)$  from the test procedure. This value of the third order moment gives information on non-Gaussianity only and is extraneous in a test of non-linearity.

The third order moment and bispectrum are sensitive to non-stationarity as well as to non-linearity. The test proposed here, and others that use the phase scrambled bootstrap method to test for non-linearity [7, 8, 9, 10, 11], have an alternative hypothesis that the data are non-linear or non-stationary. In the present paper we show how the graphical output from our test gives an heuristic indicator of non-stationarity. Using a simulation study we show how this output delineates between data generating processes that are non-linear stationary, and those that are non-linear non-stationary.

Where we refer to the “null hypothesis”, we mean the hypothesis that the given time series is linear and stationary.

## 1.2 Overview of the method

To identify non-linearity in an observed series we use an asymptotically unbiased estimate [12] of the third-order moment given by

$$\hat{\mu}(r, s) = \frac{1}{n} \sum_{t=1+|\tau|}^{n-\varphi} X_t X_{t+r} X_{t+s} \quad (-M \leq r, s \leq M), \quad (6)$$

where  $\varphi = \max(0, r, s)$ ,  $\tau = \min(0, r, s)$  and  $M$  is a truncation value ( $1 \leq M < n$ ). Due to the symmetries of the third-order moment, we need only find the estimate (6) in the region  $0 \leq r \leq s \leq M$ . We exclude the value at  $\mu(0, 0)$  and hence define  $\Delta^\# = \{(r, s); 0 \leq r \leq s \leq M, r + s > 0\}$ . The cardinality of this set is  $T^\# = (M + 1)(M + 2)/2 - 1$ .

The proposed test compares the estimated third order moment of the series with a set of limits generated from linear stationary phase scrambled bootstrap data: large differences should indicate non-linearity or possibly non-stationarity. However, recent work has shown that whilst the individual surrogate series from the phase scrambled bootstrap have desirable properties for testing non-linearity, the overall population of surrogates does not [13]. This occurs because an incorrect variance is obtained for the test statistic, and we overcome this problem by using a second bootstrap adjustment.

## 1.3 Organisation of the paper

Section 2.1 details the phase scrambling bootstrap method and Section 2.2 the proposed test of non-linearity, together with the second bootstrap adjustment mentioned above. Section 2.3 discusses some related tests. Section 3 demonstrates the large power of, and constant type I error achieved by, the new method when compared with an existing third order test using simulated linear and non-linear time series. The discussion and conclusions follow in Section 4.

## 2 Development of the method

### 2.1 Bootstrap method

We first subtract the sample mean from the series so that  $E(X_t) = 0$ . We further assume that the series is ergodic, and that the series length,  $n$ , is even. We bootstrap the original series to produce a resample  $\{X_t^*\}$  subject to the null hypothesis of linearity and stationarity. The bootstrap procedure assumes that, under the null hypothesis, we have

$$X_t = h(l_t) \quad (t = 1, \dots, n), \quad (7)$$

where  $\{l_t\}$  is a linear stochastic process and  $h$  is a static (possibly non-linear) transform. An example of a non-linear transform that results in a linear series is  $X_t = l_t^2$ .

The Amplitude Adjusted Fourier Transform (AAFT) algorithm [7] requires that  $\{l_t\}$  has a Gaussian distribution and is defined as follows.

1. Let  $\{Y_t\}$  be a sample of Gaussian white noise of size  $n$ , with  $E(Y_t) = 0$  and  $E(Y_t^2) = 1$ , reordered to the rank structure of  $\{X_t\}$ , which simulates  $h^{-1}$  by attempting to repeat the association between  $X_t$  and  $l_t$ .
2. Take the Fourier transform of  $\{Y_t\}$ , denoted as  $\{Y_t^{FT}\}$ , and extract its magnitudes  $|H_Y(\omega_j)|$  and phases  $\psi_j$  ( $j = 0, \dots, n-1$ ).
3. Generate a set of random phases,  $\{\psi_j^*\}$ , uniformly on  $[0, 2\pi)$  ( $j = 1, \dots, n/2 - 1$ ), and symmetrise the phase by setting  $\psi_0^* = 0$ ,  $\psi_{n/2}^* = \psi_{n/2}$ ,  $\psi_{n-j}^* = -\psi_j^*$  ( $j = 1, \dots, n/2 - 1$ ).
4. Back transform using the magnitude  $|H_Y(\omega_j)|$  with the bootstrap phases  $\{\psi_j^*\}$  to create bootstrap series  $\{X_t'^*\}$ .

The random phase generation preserves only the linear structure of the data so that  $\{X_t'^*\}$  is a linear surrogate of  $\{Y_t\}$ . A further step, as follows, ensures that the surrogate data have the same marginal mean and variance as the original series; indeed, it guarantees that the marginal distributions of both coincide.

5. Reorder  $\{X_t\}$  to have the same rank structure as  $\{X_t'^*\}$  to create bootstrap series  $\{X_t^*\}$ .

This is also called the rescaling algorithm [14].

It can be shown that  $\text{cov}(X_t^*, X_{t+k}^*) = \text{cov}(X_t, X_{t+k})$ , and  $\mu_X^*(r, s) = 0$  ( $r+s > 0$ ), where  $\mu_X^*(r, s)$  denotes the third order moment of the series  $\{X_t^*\}$ . Hence, in a test for non-linearity, the second order structure is preserved exactly in the bootstrap series, while at the third order, it is only the moment about zero,  $\mu(0, 0)$ , which remains fixed. In fact, as a rescaled bootstrap series is a reordered version of the original, all the central moments will be equal so,  $E(X_t^q) = E(X_t^{*q})$ , for all  $q$ . Consequently, large differences observed in the estimates of third order moments of  $\{X_t^*\}$  and  $\{X_t\}$  should be attributable to higher-order interactions in the original data, which implies non-linearity (or, as we shall argue later, possibly non-stationarity).

There are some known drawbacks to the AAFT method. The so-called flatness bias [15] is named because the autocovariance function (acv.f.) of the AAFT surrogate data tends to be closer to zero than the acv.f. of the original data (that is, the AAFT method ‘whitens’ the surrogate data, although this is not always the case [10]). Also [16] demonstrated how the autocorrelation function (ac.f.) from AAFT surrogates is biased when the function  $h$  in expression (7) is non-monotonic.

An adjustment developed by the same author known as the Corrected AAFT (CAAFT) [10] was designed to allow for a non-monotonic  $h$  in expression (7). After generating  $\{X_t^*\}$  using the rescaling algorithm, the extra steps are as follows.

- E.1) Compute the ac.f. of the observed data  $\{X_t\}$ , the rescaled surrogate series  $\{X_t^*\}$ , and the observed reordered white noise series  $\{Y_t\}$ , respectively, as  $\hat{\rho}_X(k)$ ,  $\hat{\rho}_{X^*}(k)$  and  $\hat{\rho}_Y(k)$  ( $k = 1, \dots, k_{\max}$ ) (for some  $k_{\max}$ ).
- E.2) Find the simple linear interpolation  $\hat{\theta}$  of  $\hat{\rho}_Y(k)$  as a function of  $\hat{\rho}_{X^*}(k)$ .
- E.3) Compute  $\hat{\rho}_U(k) = \hat{\theta}\{\hat{\rho}_X(k)\}$  ( $k = 1, \dots, k_{\max}$ ).
- E.4) Estimate the coefficients of an Autoregressive  $\text{AR}(k_{\max})$  model from  $\hat{\rho}_U$  using the corresponding Yule-Walker equations [17].
- E.5) Generate a model-based bootstrap time series  $\{U_t^*\}$  using the stabilized AR model from Step E.4 and randomly generated standard Gaussian i.i.d. errors [18].

- E.6) Reorder  $\{X_t^*\}$  to have the same rank structure as  $\{U_t^*\}$ , and label this series  $\{\tilde{X}^*\}$ .
- E.7) Repeat Steps E.1 to E.6 a total of  $K$  times to create  $\{\tilde{X}_t^{1*}\}, \dots, \{\tilde{X}_t^{K*}\}$ .
- E.8) Compute  $\hat{\rho}_{\tilde{X}^{1*}}(k), \dots, \hat{\rho}_{\tilde{X}^{K*}}(k)$  ( $k = 1, \dots, k_{\max}$ ), and select the optimal series as that having ac.f. closest to  $\hat{\rho}_X$  (in, say, a mean square sense).
- E.9) Use the AR model from this optimal series to generate  $A$  model-based surrogate data sets by repeating Steps E.5 and E.6.

Hence the algorithm is an  $\text{AR}(k_{\max})$  model-based bootstrap (with the additional reordering step E.6) using an AR model that has been closely matched to the second order properties of the observed data. The rationale of the above method is to find a suitable Gaussian process which under a monotonic transform gives surrogate data with approximately the same acv.f. as the original data.

## 2.2 Test statistic

For each bootstrap series we obtain its third order moment  $\hat{\mu}_{X^{*b}}(r, s)$  ( $b = 1, \dots, A$ ), and summarise these bootstrap limits over  $\Delta^\#$  (as defined in the paragraph following (6)) using the  $100\alpha\%$  percentile (for fixed  $\alpha$ )

$$\Gamma_\alpha^*(r, s) = \hat{\mu}_{X^{*(\alpha A)}}(r, s) \quad ((r, s) \in \Delta^\#) \quad (8)$$

which is calculated pointwise for each  $(r, s)$  value. The difference between the series' third order moment and lower ( $L$ ) and upper ( $U$ ) percentiles of the bootstrap data are calculated, respectively, as

$$\begin{aligned} L^*(r, s) &= \hat{\mu}_X(r, s) - \Gamma_{\alpha/2}^*(r, s) \\ U^*(r, s) &= \hat{\mu}_X(r, s) - \Gamma_{1-\alpha/2}^*(r, s). \end{aligned} \quad (9)$$

We also define

$$UL^*(r, s) = U^{*+}(r, s) + L^{*-}(r, s), \quad (10)$$

where  $U^{*+}(r, s) = \max\{U^*(r, s), 0\}$  and  $L^{*-}(r, s) = \min\{L^*(r, s), 0\}$ . Note that, in (10), we retain the information on whether the upper or lower limits were exceeded. Our test statistic is then given by

$$NL = \sum_{r=0}^M \sum_{\substack{s=r \\ s \neq 0}}^M |UL^*(r, s)|. \quad (11)$$



The test statistic is the sum of the absolute differences between the observed third order moment and the most extreme value admissible under the null hypothesis, and at significance level  $\alpha$ . Also we have information on the location of the significant values from  $UL^*(r, s)$ . This information can be used as a basis for selecting a non-linear model, such as one might construct using a second order Volterra expansion [24]. However, we suggest examining the values of  $\hat{\mu}(r, s)$  where  $UL^*(r, s) \neq 0$  when estimating the model parameters rather than the  $UL^*(r, s)$ : see, for example, some existing literature which describes the expected value of  $\mu(r, s)$  for bilinear processes [19, 20, 21, 22, 23].

The disadvantage of this test statistic in equation (11) is that the sum contains dependent terms under the null hypothesis, which complicates the construction of a parametric critical level. Also  $\text{var}\{\hat{\mu}_{X^*}(r, s)\} < \text{var}\{\mu(r, s)\}$  on the manifolds  $r = 0$  and  $r = s$ , for a white noise series using either the rescaled or CAAFT bootstrap method [13].

We overcome both the problem of the dependent summands in equation (11) and the discrepancy in variance of the third order moment by using a second bootstrap procedure. We generate  $A$  bootstrap versions of the test statistic (11) under the null hypothesis of linearity, labelled  $NL^{**b}$ ,  $b = 1, \dots, A$ . The observed test statistic  $NL$  can then be compared with the bootstrap distribution of the linear test statistics  $\{NL^{**}\}$ .

There is also a problem of obtaining a desired overall type I error, since each of the  $T^\# = (M+1)(M+2)/2 - 1$  individual tests at a  $100\alpha\%$  significance level. This is compounded by the correlation between the  $\hat{\mu}(r, s)$  values for a linear series. This problem of multiple testing is also overcome by the second bootstrap procedure. The principle is, that any bias in the limits due to the bootstrap algorithm, or increased type I error due to multiple testing, is repeated in the second set of bootstrap data which leads to a commensurate increase in the critical value.

To make the adjustment we generate a further two sets of bootstrap data  $\{X_t^{**b}\}$  ( $b = 1, \dots, A$ ), and  $\{X_t^{***b}\}$  ( $b = 1, \dots, A$ ), and calculate linear versions of the limits (9) ( $b = 1, \dots, A$ ) as

$$\begin{aligned} L^{**b}(r, s) &= \hat{\mu}_{X^{**b}}(r, s) - \Gamma_{\alpha/2}^{***}(r, s) \\ U^{**b}(r, s) &= \hat{\mu}_{X^{**b}}(r, s) - \Gamma_{1-\alpha/2}^{***}(r, s) \end{aligned}$$

and similarly

$$UL^{**b}(r, s) = U^{**b+}(r, s) + L^{**b-}(r, s).$$

We can then calculate the linear version of the test statistic

$$NL^{**b} = \sum_{r=0}^M \sum_{\substack{s=r \\ s \neq 0}}^M |UL^{**b}(r, s)|. \quad (12)$$

A critical level is then  $NL_{((1-\alpha)A)}^{**}$ , taken from the sample distribution of the values computed in (12), and hence the null hypothesis will be rejected if  $NL > NL_{((1-\alpha)A)}^{**}$ . The bootstrap  $p$ -value is  $p^{**} = \#(NL > NL^{**b})/A$ , the so-called achieved significance level [18].

In total we calculate  $3A$  surrogate series but the computation time is still reasonable. For series of length  $n = 100$  and setting  $M = 5$  and  $A = 300$  we observed a mean computation time of 30 seconds using a routine running in MATLAB version 6.0 on an SGI Origin 3000 machine.

A useful graphical tool derived from the test is a three-dimensional plot of  $UL^*(r, s)$  against  $r$  and  $s$ . This is useful for summarising the location, size and direction of those  $\hat{\mu}(r, s)$  values that lie pointwise in the bootstrap critical region, and hence gives an indication of the joint lags and magnitude of any non-linear interactions. In the results presented here we use a two-dimensional vectorised plot of  $UL(r, s)$  against  $(r, s)$ .

### 2.3 Related tests of non-linearity

The Hinich test [4] examines an estimate of the frequency-bicoherence (4) which is asymptotically complex Gaussian under the null hypothesis of linearity. The test uses the direct bispectrum estimate which is smoothed using a square window. The smoothed bispectrum is given by

$$\hat{b}^s(\omega_j, \omega_k) = \frac{1}{(M')^2} \sum_{j'=1-M'/2}^{M'/2} \sum_{k'=1-M'/2}^{M'/2} \hat{b}(\omega_{j+j'}, \omega_{k+k'}),$$

where  $H(\omega_j) = \sum_{t=1}^n X_t \cos\{-i\omega_j(t-1)\}$ ,  $\hat{b}(\omega_j, \omega_k) = \frac{1}{n} H(\omega_j)H(\omega_k)H(-\omega_j - \omega_k)$  ( $(j, k) \in \Delta$ ), and  $M' = n^c$ . A test statistic is formed from the sum of the smoothed values.

Reports of the performance of the Hinich test are mixed. A low power is quoted by [24] and a low power is demonstrated by [5] using  $n = 200$  and  $M' = 24$  for the Autoregressive Conditionally Heteroscedastic (ARCH) and Threshold Autoregressive (TAR) models defined in Table 1. Conversely [25] showed good power using a simulation study with a larger sample size

$n \geq 256$  and smaller truncation value  $M' = 0.7(n^{0.5} + 1)$  for the non-linear Bilinear and TAR models.

A test for independence based on the third order moment, called the ‘Hinich Bico-variance Test’, is discussed in [5]. The test uses an estimate of the third order moment given by

$$\hat{\mu}_H(r, s) = \sum_{t=1}^{n-s} \tilde{\varepsilon}_t \tilde{\varepsilon}_{t+r} \tilde{\varepsilon}_{t+s} \quad (0 < r < s \leq M),$$

where  $\tilde{\varepsilon}_t = (\varepsilon_t - \bar{\varepsilon}_t)/\hat{\sigma}_{\varepsilon_t}$ : the authors suggest setting  $M = n^{0.4}$  based on a simulation study. They define their test statistic as

$$NLH = (n - s)^{-1} \sum_{s=2}^M \sum_{r=1}^{s-1} \{\hat{\mu}_H(r, s)\}^2, \quad (13)$$

so they exclude the boundaries of  $\Delta^\#$  given by  $r = s$  and  $r = 0$ , which gives  $T' = M(M - 1)/2$  values in the summation. If  $\{\tilde{\varepsilon}_t\}$  has a standard Gaussian distribution then equation (13) has a Chi-squared distribution with  $T'$  degrees of freedom. To test for non-linearity using this method, a linear model is first fitted to the data and the test applied to the residuals from this fit. A drawback of this test is that by excluding the manifolds  $r = 0$  and  $r = s$ , it may miss some non-linear interactions of a strong quadratic type. However, including the manifolds  $r = s$  and  $r = 0$  in (13) includes dependent terms in the summation, arising from the  $X_t^2 X_{t+r}$  and  $X_t X_{t+r}^2$  terms. This would lead to an increased variance for the test statistic and a type I error rate for i.i.d. processes which exceeds the target level. In a simulation study [5] showed a power of 46% using  $n = 200$  and  $M = 8$  for the ARCH(4) non-linear process defined in Table 1. However, the test showed poor power for the TAR(2;1,1) model from Table 1 with a rejection rate of 10%. As we show later the largest value in the third order moment for data from such a model relates to  $\mu(0, 1)$ , which is not included in the test statistic, equation (13).

The time-bicoherence, as in (3), was proposed by [1] but the idea was not developed into a formal test of linearity. It did detail successful parametric third and fourth order tests for Gaussianity. If the null hypotheses for both of these tests are rejected then the authors suggest examining whether or not the estimated time-bicoherence is constant as a heuristic test of linearity.

The time-domain bicoherence is estimated using

$$\hat{B}_t(r, s) = \frac{\sum_{r', s'=-M}^M \hat{\mu}(r', s') \hat{\mu}(r + r', s + s')}{\sum_{j=-M}^M \hat{\gamma}(j) \hat{\gamma}(j + r) \hat{\gamma}(j + s)}, \quad (14)$$

where  $\hat{\gamma}(k) = \frac{1}{n-k} \sum_{t=1}^{n-k} X_t X_{t+k}$  is the estimate of the acv.f.. A vectorised equation (14) is plotted against lag  $\{r = 1, \dots, n-1, s = r, \dots, n-1\}$ , and whether or not the statistic is constant is determined by eye.

A similar procedure to that presented here was given in [26], but the authors only used a subset of the third order moments of the form  $\mu(r, 2r)$ .

The third order moment has been used to test for specific forms of non-linearity. The work of [27] used a matrix of the individual elements of (6) to detect non-linear phase coupling, as the rank of this matrix should equal the number of coupled sine waves in the series. A third order statistic was employed by [28] to test whether a TAR model fitted the observed data better than a linear AR model.

### 3 Simulation study

We used a simulation study to compare the power of the proposed bootstrap test with the Hinich test. We examined the processes detailed in Table 1; unless otherwise stated the series were generated using i.i.d. noise,  $\varepsilon_t \sim N(0, 1)$ . We restricted the test to short term dependence by setting  $M = 5$ .

#### 3.1 Results

The achieved significance levels are shown in Table 2 for  $n = 100$  and  $200$ , and using  $M = 5$ ,  $A = 500$  and  $\alpha = 5\%$  for 500 simulations of each data type. We used both the rescaled AAFT and CAAFT bootstrap algorithms. The results are compared with the test from [4] which uses a smooth estimate of the frequency-bicoherence and a parametric critical level based on the asymptotic distribution of the bispectrum under the null hypothesis of linearity. We used  $M' = 12$  and  $24$ , for  $n = 100$  and  $n = 200$ , respectively, for the Hinich test, as used by [5] who also provide the code to run the test. As noted earlier if the series is linear then the bicoherence is constant and this is tested by examining the range of observed bicoherence values. In this study the inter-decile range (IDR) performed better in terms of observed rejection rates than the inter-quartile or 80% ranges which are given as possible alternatives. Hence it is

Table 1: Linear, non-linear and non-stationary data generating processes

Label	Model	Type <sup>†</sup>
AR(0.4,1)	$X_t = 0.4X_{t-1} + \varepsilon_t$	LS
AR(0.99,2)	$X_t = 1.9X_{t-1} - 0.9001X_{t-2} + \varepsilon_t$	LS
BL(0.4,1,1)	$X_t = 0.4X_{t-1}\varepsilon_{t-1} + \varepsilon_t$	NLS
BL(-0.3,2,3)	$X_t = -0.3X_{t-2}\varepsilon_{t-3} + \varepsilon_t$	NLS
BLE(0.4,1,2)	$X_t = 0.4\varepsilon_{t-1}\varepsilon_{t-2} + \varepsilon_t$	NLS
TAR(2;1,1)	$X_t = \begin{cases} -0.5X_{t-1} + \varepsilon_t, & X_{t-1} < 0.5 \\ 0.4X_{t-1} + \varepsilon_t, & \text{otherwise} \end{cases}$	NLS
ARCH(4)	$X_t = \varepsilon_t \sqrt{h_t}$ $h_t = 0.000019 + 0.846\{X_{t-1}^2 + 0.3X_{t-2}^2 + 0.2X_{t-3}^2 + 0.1X_{t-4}^2\}$	NLNS
Quadratic phase	$X_t = \sum_{j=1}^3 \cos(\omega_j t + \phi_j) + \varepsilon_t$ , $\omega_1 = \omega_2 = \pi/3$ , $\omega_3 = \pi$ , $\phi_1, \phi_2 \sim U[0, 2\pi]$ , $\phi_3 = \phi_1 + \phi_2$	NLS
Trilinear(0.1)	$X_t = 0.1X_{t-1}X_{t-2}X_{t-3} + \varepsilon_t$	NLS

<sup>†</sup> LS=Linear stationary, NLS=Non-linear stationary, NLNS=Non-linear non-stationary

AR=Autoregressive, BL=Bilinear, BLE=Bilinear Error, TAR=Threshold Autoregressive, ARCH=Autoregressive Conditionally Heteroscedastic

the IDR results that we show in Table 2, labelled as ‘Hinich’. It is important to note that we are comparing a non-parametric time-domain test in the present paper with a parametric frequency-domain test.

Figure 1 plots the vectorised mean and standard deviation  $UL(r, s)$  values for  $(r, s) \in \Delta^\#$  from the 500 simulations for the non-linear BL(0.4,1,1), BL(-0.3,2,3), BLE(0.4,1,2), TAR(2;1,1), ARCH(4) and quadratic phase processes given in Table 1. Figure 2 shows the mean rejection rate from 200 simulations over an increasing sample size  $n = 50, 60, \dots, 250$ , for the linear AR(0.4,1) and a non-linear BL(0.4,1,1) processes using the CAAFT bootstrap with  $A = 200$ .

## 4 Discussion

We first note that we investigated the random number generator for non-linearity using the present methodology, as shown in Table 2 under the heading “Gaussian”: clearly, the achieved significance level is very close indeed to 5%, so we are satisfied that our subsequent simulations have integrity.

We chose values of  $M$  and  $M'$  so that comparisons with results in previous papers would have a common basis. Unless there were strong long memory in the data, the form of (14) suggests that the test should be reasonably robust to the choice of  $M$ , provided it is not too small, as discussed below.

In this simulation study the present method outperformed the Hinich test, in that its rejection rates were higher for all the non-linear processes, and were closer to the desired 5% level for the linear processes. The exception was the non-linear Trilinear(0.1) process to which neither method was sensitive but the Hinich test gave a slightly higher rejection rate for  $n = 100$ . We would expect neither test to be sensitive to this process as the non-linearity is in the fourth order.

We claim there are three reasons why both bootstrap tests were superior the Hinich test. Firstly, by estimating the variance of the third order moment using the non-parametric bootstrap, the proposed method is relatively unaffected by the previously cited problems with the parametric assumptions made by the Hinich test, and stability of the results according to sample size appears to be achieved relatively quickly. Secondly, our test excludes the value at  $\mu(0, 0)$ , the Hinich test does not. This value of the third order moment confounds information on non-linearity. Third, our test examines a truncated region of the third order moment bounded by  $M$ . The Hinich test uses the direct estimate of the bispectrum which is equivalent to the double Fourier transform of the entire third order moment ( $M = n - 1$ ). As shown in Figure 1, the region of the third order moment with large values may be small depending on the type of non-linearity present. Therefore by using an estimate that averages over the entire third order moment the Hinich test has decreased sensitivity to these localised significant values.

The CAAFT bootstrap method performed slightly better than the uncorrected AAFT. Both methods have achieved significance levels close to the desired value of 5% for the linear processes, although for the linear process with  $t$ -noise the AAFT method gave a high rate (8.6%) for  $n = 100$ . For the same process of length  $n = 200$  the CAAFT gave a low rate (1.8%). The methods have comparable and good rejection rates for the non-linear

processes except for the quadratic phase model where the CAAFT method performs almost twice as well as the AAFT. The CAAFT method provides a better match to the ac.f. of the original data for this process than the AAFT. By correctly explaining more of the variance in the second order of the data the surrogates from the CAAFT method are better able to detect departures in the third order.

Both bootstrap methods gave a reasonable (close to 5%) rejection rate for the linear unstable AR(0.99,2) process, whereas the parametric Hinich test had a very high type I error rate: 93% for  $n = 200$ . The Hinich test also gave an inflated type I error for the AR(1,0.4) process using  $t$ -noise whereas the results from both bootstrap tests were reasonably close to 5%. As well as being insensitive to a trilinear process the proposed test would be unable to detect third order non-linear interactions that existed outside  $\Delta^\#$ . This can be overcome by setting  $M$  to the maximum value of  $n - 1$  as well as repeating the test for a range of  $M$  values. Also plots of the test statistic  $NL$  against  $M$  are useful for detecting the location of the non-linear interactions [29].

For both tests the BL(-0.3,2,3) process gave a lower rejection rate than the BL(0.4,1,1). This is somewhat due to the smaller non-linear parameter (-0.3 compared to 0.4), but also the theoretical third order moment of a BL(-0.3,2,3) process has only one large value at  $\mu(1,3)$  with all other values being zero [22]. A BL(0.4,1,1) process has large values at both  $\mu(0,1)$  and  $\mu(1,1)$  (see Figure 1). The modal absolute maximum for the BL(-0.3,2,3) process over the 500 simulations was at  $UL^*(1,3)$  reflecting the expected third order moment. Note also the negative mean value for  $UL^*(1,3)$  in Figure 1 reflecting the negative model parameter.

In Figure 1 the  $UL^*(r,s)$  values were unimodal over  $\Delta^\#$  for the non-linear stationary processes: BL(0.4,1,1), BL(-0.3,2,3), BLE(0.4,1,2), TAR(2;1,1) and quadratic phase. Also over the 500 simulations the large values within these process types were fairly consistent. This is reflected by the relatively small values of the standard deviation, with larger variation at the significant interaction terms. For the non-stationary ARCH(4) process the patterns were not consistent between simulations. The ARCH(4) process had significant  $UL^*(r,s)$  values over a number of  $(r,s)$  coordinates. The largest observed values over the 500 simulations were at  $UL(2,2)$  and  $UL(2,3)$  and the next largest within  $r \leq s \leq 4$ . (This may be due to the order of the ARCH data being four.) This multimodality in the ARCH(4) results can be allied to that observed by [14] in the AAFT surrogates of non-stationary data. The non-

stationarity in the ARCH series causes a large variability in the significant third order moments found over the 500 simulations. Hence multimodality in a plot of the  $UL^*(r, s)$  can be used as an heuristic indicator of non-stationary.

Figure 2 shows a relatively constant and close to 5% type I error rate for the proposed test using a linear AR(0.4,1) process, and an improving power for a non-linear BL(0.4,1,1) process over increasing  $n$  using CAAFT bootstrap surrogates.

It is in order to make some comments about interpretations of non-linearity and non-stationarity suggested by the present test.

In respect of non-linearity, while indeed third-order quantities capture quadratic curvature in a time series, such as through a Volterra expansion, one needs to consider what the presence of such a term *alone* might imply. This reduces to a typical regression problem; for example, a polynomial regression of a response on terms of up to power  $m$  will render a different model, and possibly with substantive differences among collections of significant terms, if one were to perform a polynomial regression using the same data of up to power  $m - 1$ . In considering only the third order moment, we can not hope to identify all sources of non-linearity in the time series: there is an uncountably infinite number of “types” of non-linearity. Thus, it is plausible that significant results from the present bootstrap test will identify non-linearity beyond quadratic which has become confounded with the quadratic term. Of course, we concede that the opposite may also occur, in that some kinds of non-linearity not of a quadratic type may go undetected, such as illustrated in the case of the Trilinear process in our numerical study.

In respect of non-stationarity, we do not attempt to deal with the issue of defining and estimating the third-order moment in the presence of non-stationarity. However, the results of our fairly extensive simulation study have shown that non-stationarity may contain specific signatures in terms of the outcomes of the test, and thus the user should be alert to the fact that a significant departure from the null hypothesis may admit non-stationarity as well as, or even instead of, non-linearity.

Further simulation studies, applications to real data sets and discussion is given in [29]. For example, the test has been applied to hydrological data (flooding levels of a river), some specific examples of which are typically believed to contain non-linearity through long memory, and to financial data (exchange rates). The results support some choices of parametric non-linear models used by other researchers to model these data. There is considerable discussion worthy of even these two applications alone, which is beyond the



scope of the present paper, and which constitutes current work in progress of the authors.

There would, of course, be much interest in applying the test to deterministic, chaotic systems. There is a number of theoretical issues, such as interpreting the third order moment as an expectation with respect to the (absolutely continuous) invariant measure of the dynamical system in question, should such a measure exist. Again, this is beyond the scope of the present paper.

Table 2: Comparison of achieved rejection rates (%) using the Hinich bispectrum test and the present bootstrap time-domain test for non-linearity over 500 simulations

Series	Type	Hinich	Bootstrap	
			AAFT	CAAFT
$n = 100$				
Gaussian	LS	6.0	5.8	4.4
AR(0.4,1)	LS	5.0	4.8	4.2
AR(0.4,1) <sup>†</sup>	LS	27.4	8.6	6.0
AR(0.99,2)	LS	25.0	9.4	3.6
BL(0.4,1,1)	NLS	19.0	89.4	85.0
BL(-0.3,2,3)	NLS	6.8	31.6	30.2
BLE(0.4,1,2)	NLS	8.2	41.2	40.4
TAR(2;1,1)	NLS	9.6	41.0	38.0
ARCH(4)	NLNS	40.2	86.4	82.8
Trilinear(0.1)	NLS	7.0	5.2	5.4
Q-phase	NLS	15.8	19.2	41.2
$n = 200$				
Gaussian	LS	2.0	4.4	3.6
AR(0.4,1)	LS	1.6	4.8	4.8
AR(0.4,1) <sup>†</sup>	LS	16.2	5.4	1.8
AR(0.99,2)	LS	93.0	8.0	3.4
BL(0.4,1,1)	NLS	17.4	99.0	99.0
BL(-0.3,2,3)	NLS	4.6	67.6	65.4
BLE(0.4,1,2)	NLS	5.2	81.2	79.2
TAR(2;1,1)	NLS	8.0	79.2	79.0
ARCH(4)	NLNS	46.8	96.0	95.2
Trilinear(0.1)	NLS	2.0	7.4	6.6
Q-phase	NLS	2.8	29.4	71.0

LS=Linear stationary, NLS=Non-linear stationary, NLNS=Non-linear non-stationary.

<sup>†</sup> Errors from a zero mean  $t$ -distribution with 2 degrees of freedom.

Rejection rates for LS data  $\leq 9.4\%$  for AAFT method and  $\leq 6.0\%$  for CAAFT method. Both bootstrap tests have higher rejection rates than the Hinich test for all NLS or NLNS processes bar the fourth order trilinear process.

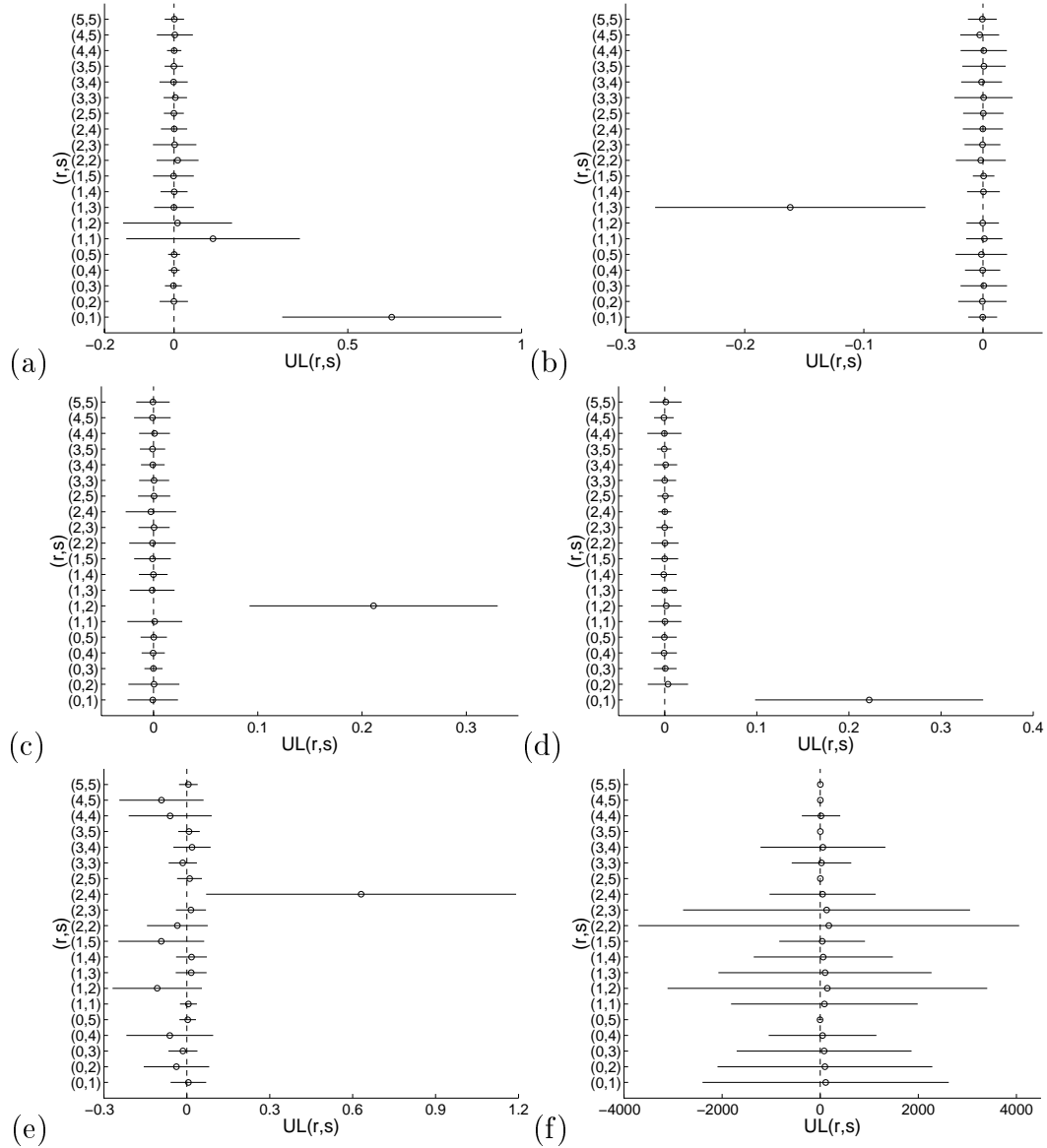


Figure 1: Mean and standard deviation of  $UL(r, s)$  values from simulation for non-linear processes.

(a) BL(0.4,1,1), (b) BL(-0.3,2,3), (c) BLE(0.4,1,2), (d) TAR(2;1,1), (e) quadratic phase, (f) ARCH(4).

Results from using CAAFT bootstrap method and  $n = 200$ . A circle represents the mean  $UL(r, s)$  value and lines extend to one standard deviation either side of the mean. Dotted line at  $UL(r, s) = 0$  indicates no non-linear or non-stationary interaction. Note the scales on the  $x$ -axes are not consistent across panels and the scale for the ARCH(4) results is much larger. Non-linear stationary processes, panels (a) to (e), have distinct large mean values at  $(r, s)$  pairings that correspond to the data generating process. The non-linear non-stationary ARCH(4) series, panel (f), has several large  $UL(r, s)$  values, which also have a large variance.

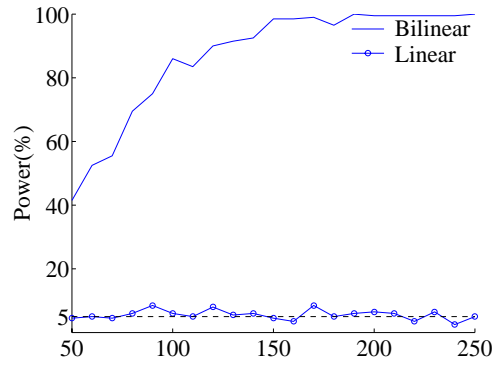


Figure 2: Achieved significance level (Power %) against sample size ( $50 \leq n \leq 250$ ) using the CAAFT bootstrap test of non-linearity for simulated linear AR(1) and non-linear bilinear BL(0.4,1,1) processes

200 simulated series created for each sample size, individual series tested using  $A = 200$  bootstrap surrogates.

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